To recent 150th Anniversary of Sofia Kovalevskaya (1850-1891): her scientific legacy in Celestial Mechanics of equilibrium figures of fluid mass in axial rotation.

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POLYAKHOVA, Elena Nikolaewna with the head of the Celestial Mechanics Department of University of St. Petersburg, Professor Konstantin Kholshevnikov, in front of the statue of Sofia Kovalevskaya**.**

Elena N. Polyakhova, born in 1935 in Leningrad, graduated from Astronomy Department, Faculty of Mathematics and Mechanics of Leningrad (now St. Petersburg) University in 1957. She works in Astronomical Department, Celestial Mechanics Division, as assistant Professor now as associated Professor, Phd.

The scope of her interests concerns to Celestial Mechanics, Astrodynamics (Space Flight Dynamics), Solar Sailing Theory, History of Natural Sciences (Mechanics, Astronomy, Astrodynamics, biographies and scientific legacies of Russian scientists, especially of Sofia Kovalevskaya, Michael Ostrogradsky, Alexander Lyapunov, history of St-Petersburg scientific schooles in Classical and Celestial Mechanics).

She published about 150 scientific papers, 3 books. Author of the first in the world monography on Solar Sailing in Space: "Spaceflight with Solar Sail" Moscow, 1985.

Elena Polyakhova is honoured by name of minor planet (asteroid): minor planet (NMP)"4619 Polyakhova" is named after her.

Content

I. Primary events and dates in S.Kovalevskaya's life and scientific work

In January 2000, the world's scientific community celebrated the 150th Centennial Jubilee Anniversary of Sofia Wasil'evna Kovalevskaya - outstanding Russian scientist-mathematician, writer, publicist, the world's first female professor, and elected Correspondent-Member of Academy of Sciences in St. Petersburg, Russia. Professionals in the areas of Mathematics, Mechanics, Physics and Astronomy, History and Literature acknowledge her heritage. As for her remarkable scientific legacy, S.V. Kovalevskaya had published just 9 works: 3 of pure mathematical nature and 6 on the mathematical applications to the problems of Celestial Mechanics (1 article), Physics of crystals (2 articles) and Classical Mechanics of rigid body rotational motion (3 articles) (see Appendix 1).

With connection to this memorable date in the history of science, let us recall major events and dates of S.Kovalevskaya's (nee Korvin-Krukovskaya) life and scientific activity (some references concerning her life presented in Appendix 2):

1850: 3(15) of January, born in Moscow.
1853-1858:lived with her parents in Kaluga.
1858: moved to country estate of Palibino, not far from the city of Velikiye Luki, now Pskov region of Russia.
1866: began to study mathematics with N.A. Strannolyubski during the family travels to St. Petersburg.
1867: traveled with family to Germany and Switzerland.
1868: 15(27) of September, marriage with Wladimir Kovalevsky (1842-1883), moved to St. Petersburg to study science.
1869: trip to Heidelberg to study with G. Kirchhoff (1824-1887) and H. Helmholtz (1821-1894); her trip to London.
1870: moved to Berlin and began private mathematical studies with Karl Weierstrass (1815-1897), the greatest analyst of his time.
1871: stayed in Paris during the city siege and the Paris Commune Days; helped her brother-in-law Victor Jaclaire to escape from prison.
1872-1874: worked in Berlin on Abelian integrals Mathematics and Saturn's rings stability Mechanics.
1874: in her absence, the University of Goettingen awarded Kovalevskaya a Ph.D. in Mathematics and Master of Fine Arts with support of Karl Weierstrass for the cycle of three works over the period of 1870-1874: I. "To the theory of partial differential equations" (her first work on pure Mathematics first published in German as her dissertation "Zur Theorie der partiellen Differentialgleichungen", Inaugural Dissertation. 1874, and then republished with the same title in Journal fuer die Reine und Angewandte Mathematik. Berlin. 1875. V.80. P.1-32), II. "About reduction of same class third rank abelian integrals to elliptic integrals" (her second work on pure Mathematics, will be published only in 1884), III. "Addendum and remarks to Laplace's Saturn's rings shape research" (her first and only single work in Celestial Mechanics, will be published later, only in 1885), returned back to Russia.
1874-1877: literary-publicist work in St. Petersburg; worked as a journalist in Suvorin's newspaper "New Time"; meetings with Mendeleev, Setchenov,

Botkin, Butlerov, Chebyshev, Stoletov, Turgenev, Dostoevskiy, etc.
1878: birth of her daughter Sofia Wladimirovna Kovalevskaya (1878-1952).
1879: returned back to scientific work, with support from mathematician P.L. Chebyshev (1821-1894) gave a talk at the 6th Russian Naturalists' Convention in St. Petersburg on properties of Abelian integrals.
1880: moved to Moscow and was elected to be a member of Moscow Mathematical Society.
1881-1883: lead scientific work in Berlin and Paris on mathematical description of refraction of light in crystals.
1883: death of her husband, W.O. Kovalevsky; returned to Russia; gave a talk at the 7th Russian Naturalists' Convention in Odessa on refraction of light in crystals; moved to Stockholm as private assistant professor in newly founded Stockholm University by invitation of Professor G.M. Mittag-Leffler (1846-1927); published article IV entitled "About light refraction in crystals" (her first work in Physics, published in German as "Ueber die Brechung des Lichtes in kristallinischen Mitteln." Acta Mathematica. Stockholm. 1883. V.6. P.249-304),
1884: 30th of January (11th of February), gave her first lecture in Stockholm University; was appointed ordinary Professor of Stockholm University and invited to join editorial board of the journal "Acta Mathematica", founded by G.M. Mittag-Leffler and edited in Sweden (as is known, Mittag-Leffler began discussions about the journal in 1881, the first issue of Acta was published in 1882); published article II (see above) in German as "Ueber die Reduktion einer bestimmten Klasse Abel'schen Integrale 3-en Ranges auf elliptische Integrale." Acta Mathematica. Stockholm. 1884. V.4. P.393-414); published article V "About propagation of light in crystalline medium" (her second work on Physics of crystals, published in French as "Sur la propagation de la lumiere dans un milieu cristallise'." Comptes Rendus Hebdomadaires de Seances de l'Academie des Sciences. Paris. 1884. V.98, P.356-357, and then published again in Swedish as "Om ljusets fortplantning uti ett kristalliniskt medium." Ofversigt uf Konigl. Vetenskapsakademiens Foerhandlingar. Stockholm. 1884. V.41. P.119-121).
1885: published article III (see above) in German as "Zusaetze und Bemerkungen zu Laplace's Untersuchung ueber die Gestalt der Saturnringe" (Astronomische Nachrichten. Kiel. 1885. Bd.111. N.2643. P.37-48),
1886: published essay "Remembrances of George Elliott" in journal "Russian Thought" (1886, N.6), based on her visits with the English writer in 1869 in London and on their long-term correspondence.
1887: worked on drama "Fight for Happiness," co-written with Swedish writer Anne-Charlotte Leffler-Edgren (later the play was staged in Russia).
1888: worked on the topic of rigid body rotation; was awarded Borden's Prize for the work VI that made her famous: "Problem of rigid body rotation around fixed point" (her first work on Classical Mechanics) by the Paris Academy of Sciences.
1889: published the article in French as "Sur la probleme de la rotation d'un corps solide autour d'un point fixe," (Acta Mathematica. Stockholm. 1889. V.12. P.177-232); published literary essay about N.E. Saltykov-Shedrin for a French newspaper; after resolution

of the issue of female acceptance in "Academia" by St. Petersburg Academy of Sciences on 14(26) of December 1890 and with active support of russian Academicians Chebyshev, Imshenetsky and Bunyakovsky, on 7 (19) of November 1889, Kovalevskaya was elected to be a Correspondent-Member of Mathematics and Physics Division of St. Petersburg Academy of Sciences.

1890: published literary essay "Three days in peasants' university of Sweden"; worked on the book "Reminiscence of childhood", short story "Nihilist" (unfinished), and on the novel "Nihilstka" (female-nihilist), this last publication was not allowed in Russia (the novel only saw light in Russia in 1928), when it was published in Sweden very soon under the title "Vera Voronzoff"; Kovalevskaya was awarded the Prize of Sweden Academy of Sciences for the work VII "About the property of differential equations system, defining the rotation of rigid body around fixed point" (her second work on Classical Mechanics, which was the essential development and continuation of the work VI); article VII was published in French as "Sur la propriete' du systeme d'equations differentielles qui definit la rotation d'un corps solide autour d'un point fixe." (Acta Mathematica. Stockholm. 1890. V.14. P.81-93); worked on article VIII about one private case of problem of rigid body rotation around fixed point when integration is done by the means of ultra-elliptical functions of time (her third work on Classical Mechanics; the continuation and development of works VI and VII); article VIII was published in French as "Memoire sur un cas particulier du probleme de la rotation d'un corps pesant autour d'un point fixe, ou l'integration s'effectue a l'aide de fonctions ultraelliptique du temps" (Memoirs presentes par divers savants etrangers a l'Academie des Sciences de l'Institut National de France. Paris. 1890. V.31. P.1-62).

1891: book "Reminiscence of childhood" was published in Russian journal "European Bulletin" and in Swedish journal entitled as "Systrarna Rajevsky"; Kovalevskaya's work X about one Brunce's theorem from theory of potential (her third work on pure mathematics) was published in French as "Sur un theoreme de M. Bruns" (Acta Mathematica. Stockholm. 1891. V.15. P.45-52) soon after her death.

1891: death of Sofia Kovalevskaya on January 29th (February 10th) 1891 in Stockholm where she was buried on Karolinian cemetery.

Sofia Kovalevskaya is honoured by names of lunar crater and minor planet (asteroid): minor planet (NMP)"1859 Kovalevskaya" discovered in 1972, September, 4th, in Nautchny-Observatory in Crimea (former USSR), and preliminary numbered as 1972 RS2, is named by its discoverer L.V. Zhuravleva after this famous russian woman.

II. About Kovalevskaya's scientific legacy in Celestial Mechanics (her results in Celestial Mechanics of equilibrium figures of fluid masses in axial rotation)

As is known, S. Kovalevskaya got the fundamental results in the theory of partial differential equations (the classical Cauchy-Kovalevskaya's theorem). These her results were the first in the series of works which were completed in 20th century only. As usually, the result of Kovalevskaya in Classical Mechanics is widely known as concerned to her research on the rotation of a rigid body about a fixed point. She carried out this research after her appointment to a professorship in Stockholm in 1880s.

In general, the equations of motion of such a body have three degrees of freedom, the orientation of the body being specified by three Euler's angles. Only two completely integrable cases were known at that time. In the first case corresponding to a body freely tumbling without feeling gravity both angular momentum vector and energy are conserved. This leads to the so-called Euler's case of regular precession. The case was proved, described and partly published by L. Euler (1707-1783) in 1750 and 1758 but firstly published completely in 1765 (Euler L. Du mouvement de rotation des corps solides autour d'un axe variable. Memoires de l'Academie des Sciences de Berlin. 1765. V.14. P.154-193). In the second case, the body has the symmetry of a spinning top, the system is still integrable. This leads to the well-known Lagrange's case of non-regular precession of heavy gyroscope. The theory was developed by J.L. Lagrange (1736-1813) during the preparation of the second edition of his "Analytical Mechanics" (published firstly in 1788). After Lagrange's death this theory was included by his colleagues into the second book of the second edition of Lagrange's "Analytical Mechanics" (J.L. Lagrange. Mecanique Analytique. 2-ieme Edition. Paris. Courcier-Imp. Libr. 1815. V.2. Section 9).

Kovalevskaya found a third completely integrable case, in which the moments of inertia of the body are related by a particular way and when integration is done by means of hyperelliptic functions of time. Her first article on it (see above article VII) won the Bordin Prize's of Paris Academy of Sciences in 1888, on which occasion the Prize Commitee doubled the usual prize money, in recognition of an unusual achievment. She became the leading woman mathematician, "Princess of Science", earning the respect and confidence of many scientists. She had published in 1880s three articles about Classical Mechanics of rigid body rotation about a fixed point (see above articles VI, VII, VIII). Her second article on it (VII) won the Prize of Sweden Academy of Sciences in 1889. These three her papers were her last articles in 1880s, Kovalevskaya's last article (IX) was published already after her death, in 1891.

Unsimilarly to these usually and widely known remarkable and famous results of Kovalevskaya, it might be interesting to remember to one of her early articles (see above articles I, II, III). These three manuscripts were accomplished during her 4-year staying and study in Berlin under K. Weierstrass's scientific guidance. Sonya was scarcely twenty in 1870 when she met Karl Weierstrass, the greatest analyst of his time and a master expositor. He held a chair in Mathematics at the Royal Polytechnical School in Berlin, then at Berlin University, to which Sonya had come to study from Heidelberg. Sonya was not permitted to attend Weierstrass's public lectures, but he arranged to tutor her privatly. Sonya's first three manuscripts were sent (with the presentation written by K. Weierstrass) in 1874 to Goettingen University. Sonya have succeded to be awarded there by her first scientific degree (see above).

In her single astronomical paper published in 1885 only (see above article III) but prepared long before, in 1874, Kovalevskaya has considered, (according to P.S. Laplace's theory of Saturn rings structure), the profile shape and equilibrium stability of a single gravitating ring (fluid or monolite) around an attractive centre. She presented the development and essential expansion of Laplace's results up to second order approximation in the determination of ring gravitational potential.

This her paper is mentioned and reffered very seldom and commented else seldomer, being considered mainly as a nice and original example of the successful solution done by use of hyperelliptic functions effectively applied to a some formal problem of gravitational potential theory. It is partly true as concerned to Saturn rings real structure but it is wrong as concerned to the theory of fluid mass stability in general. Indeed, these her results were not applied by astronomers at all because since 1859 J.C. Maxwell's theory of Saturn rings with account of their particles differential keplerian orbiting was already successfully applied in Planetary Celestial Mechanics dealing with Saturn rings observations and researches.

Indeed, this article dealing with Saturn rings fluid or solid structure (after P.S. Laplace) contradicted obviously to Maxwell' theory. Nevertheless, this Sonya's single astronomical paper is now not only of historical-memorial interest but of great importance for the modern theory of rotating celestial bodies potentials. This paper turned out to be one of basic works in the theory of figure shapes of rotating fluid mass together with suitable famous results of many others scientists later. This publication entered into the brilliance chain of scientific results in this branch of Celestial Mechanics. Her method of hyperelliptic functions application for determination of potential of rotating body was investigated and higly estimated later by H. Poincare', F. Tisserand, A. Lyapunov, P. Appell, H. Lamb (see below References List). For example, already in 1891 Francois Tisserand methodically exposed Kovalevskaya's results in his book "Traite' de Mecanique Celeste" together with conventional results of Laplace, Maxwell and Poincare'.

We would like to go deeper into the history and mathematical foundations of a very limited but important problem of the field of Celestial Mechanics: fluid rotating celestial body equilibrium figures and their stability. Much of this history did not seem to be widely known especially as to Kovalevskaya's results in this problem. To investigate equilibrium figures of fluid mass, was proposed to Sonya by russian mathematician P.L. Chebyshev in 1860s in St. Petersburg, long before her depart to abroad but she had shown no interest in this problem at that time. As is known, Chebyshev discussed mathematical problems with Sonya during her study in Germany, in his letters. We suppose to be very possible that the problem of Saturn ring shape appeared in Sonya's mind namely in connection with early Chebyshev's ideas in 1870s again, because Chebyshev as professor of St. Petersburg University widely dealt with astronomy and geodesy, being interesting in ring-like structures too.

As to personal meetings or letters between Weierstrass und Chebyshev, there are no exact informations, no letters in archives. Perhaps, Sonya's letters helped their scientific contacts. For example, in Weierstrass's letter dated 12 January 1875 addressed to Sonya after her return to Russia we read: "Du schreibst mir vor einiger Zeit, dass Tschebyscheff liebe, Dir Fragen in Betreff der "Integration elliptischen Differentiale mittels Logarithmen" vorzulegen. Dies hat mich veranlasst, meine alte Arbeit ueber den Gegenstand wieder aufzunehmen, um Dich in Betreff desselben - unter Anwendung der Dir gelaeufige Methoden und Bezeichnungen - en fait zu setzen". As connect to Sonya's interest to rings thematics, Weierstrass wrote to Paul Du Bois-Reymond in his letter dated 12 December 1874 (Weierstrass, 1923. S.204) that this work was made fully by her own initiative ("Diese

Arbeit war ganz aus Kowalewskajas eigener Initiative hervorgegangen").

As is known, several years later, in 1881, Chebyshev proposed to his pupils, both Y.I. Zolotarev (1847-1878) and A.M. Lyapunov (1857-1918) to research this problem in order to develop the former results of both Maclaurin and Jacobi about equilibre ellipsoids. The so-called Chebyshev's problem can be formulated as follows: to prove the existence of some new figures of equilibrium near to two kinds of Maclaurin's ellipsoids. Only Lyapunov liked this problem and he accomplished successfully its first approximation in 1884. Simultaneiusly, in 1880s, H. Poincare' began to solve this problem too. Poincare' has found some new figures one year later than Lyapunov, in 1885, but not especially exactly. Lyapunov proved finally this result exactly in 1900s only.

As is known, P.L. Chebyshev played the important positive role for Sonya's success in Paris Academy of Sciences also. He was elected to correspondent-member of Paris Academy in 1860, as academician-member in 1874. He had the wide scientific letters with Josef Bertrand and Charles Hermite, both being consequently Presidents of Paris Academy of Sciences of French Institute. As to Hugo Gylden, Sonya's colleage in Stockholm, mathematician and astronomer, he belonged also to this mathematical french school where Chebyshev's ideas were widely known and intensively discussed. Hugo Gylden connected often with Henri Poincare' in mathematics problems, he knew about both Poincare's interest to Chebyshev's problem and about Lyapunov's recent results on it. Lyapunov published his first result in 1884 in his Magister Dissertation, Poincare' - in 1885 (Liapunov's result was written in Russian, being translated into French in 1904 only, in Toulouse.)

To our opinion, namely H. Gylden advised Sonya in 1880s to publish her forgotten manuscript of 1874 because of arising interest of mathematicians in 1880s in St. Petersburg and in Paris to equilibrium figures theory.

In order to go deeper into the results of Kovalevskaya let us consider shortly the history of Saturn rings investigations.

III. Short history of Saturn's rings discovery and early primary researches.

1610-1616 Galileo Galilei (1564-1642) observes two faint objects links and rights from Saturn.

1655 Christian Huygens (1629-1695) observes the single bright ring around Saturn.

1675 Gean Domenico Cassini (1625-1715) observes the gap (division) in this single ring (so-called Cassini's gap).

XVII-XVIII centuries Development of theories of fluid or monolite solid single ring with one gap in it.

1755 Immanuel Kant (1724-1804) proposes the first theory of the differential rotation of particles in viscosity-resistant disc-like ring of Saturn in framework of his remarkable cosmogonic theory concept [Kant, 1755]. Being written as a some philosophical thesis Kant's book was practically unknown to astronomers till beginning of XIX century when in 1805 Alexander Humboldt mentioned this book as an outstanding achievement of natural science .

1789-1798 Pierre Simon Laplace (1749-1827) publishes his theory of Saturn rings system structure [Laplace, 1789,1796] in framework of his famous cosmogonic theory concept.

1. Laplace proposes the model of rings structure as the system of many concentric narrow thin rings, fluid (iced) or monolite (solid), every ring being of its own rotational velocity "from ring to ring".
2. I. Kant's cosmogonic theory is unknown to Laplace at all.
3. Laplace studies the stability of a single narrow monolite ring and proves both its instability at its constant thickness and alternatively its stability at variable thickness (cross-section). Laplace insists the model of Saturn rings as a system of distinct monolite narrow rings, every ring being of variable cross-section.
4. Then, Laplace derives the expression for the gravitational potential of a fluid circumplanetary ring to an outer point of its surface. His result obtained analytically shows that the fluid rotating ring cross-section by a plane containing the planet rotation axis is of elliptical shape, the ellipse major axis being located in the planet equatorial plane. The cross-section of this thorus-like ring was named later after P.S. Laplace as "Laplace's ellipse".

1859 James Clerk Maxwell (1831-1879) writes in Cambridge University his manuscript "On the stability of the motion of Saturn's ring" published as the book and then later as the article [Maxwell, 1859].

1. Maxwell uses and continues Laplace's ideas of rings system but he considers every narrow ring as a system of discrete particles, each of them moving according to Kepler's Laws. It was the first idea of the ring differential rotation "from particle to particle".
2. Maxwell proves that every narrow ring can be stable only when contains the heavy satellite inside it, mass of this satellite being

4.5 times as total mass of the ring.
3. Maxwell's publication can be regarded as the first work on collective processes physics in history of science.

1870-1874 Sofia Kovalevskaya (1850-1891) finds more general solution of Laplace's problem of stability of a three-dimensional thorus-like ring in rotation.

1. She studies during her staying in Berlin the famous Laplace's book (in five volumes) "Traite' de Mecanique Celeste" [Laplace, 1796]. Very possible that this book was already partly known to Kovalevskaya else in Russia because her Great-Grand-Father F.I. Schubert (1758-1825), russian academician-member of St. Petersburg Academy of Sciences, mathematician and astronomer, author of the first in Russia textbook in Celestial Mechanics (Traite' d'Astronomie Theorique, 1822), was the letter-correspondent of P.S. Laplace during many years .
2. Kovalevskaya is interesting in this branch of Celestial Mechanics for which solutions of the hyperelliptic functions theory may be successfully applied. We suppose (see above) that her interest to rotating bodies potential and equilibrium stabilty was influenced by P.L. Chebyshev personnally in St. Petersburg long before her staying in Germany.
3. Kovalevskaya studies Laplace's results in planetary rings stability and finds out that his conclusion about the elliptic shape of a rotating thorus-like ring cross-section is not enough exact and can be consider as the first order solution only. This her estimation was true because Laplace's theory used a simple approximation of a thorus volume element by an infinite cylinder element.
4. She tries to resolve this problem in more general form: the rotating massive thorus gravitational potential is calculated by use of hyperelliptic functions, the general stability condition is taken as a "non-escape" staying of a particle on the rotating ring surface. Central planetary gravitation, ring gravitation and rotation frame inertia are taken into account.
5. Kovalevskaya finds that a particle stays stable on the three-dimensional ring surface when this ring cross-section has a not elliptic (after Laplace) but a some oval shape. The equation of a boundary curve of this cross-section was derived analytically and exposed as a some infinite convergent serie expansion. The first order term in Kovalevskaya's solution turned out to be "Laplace's ellipse" again, the terms of higher orders can be determined up to arbitrary desired order. The lengthes of major and minor axes of the resulting cross-section oval shape are the same as for "Laplace's ellipse". The orientation of the major axis in the planet equatorial plane can be altenatively opposite: the ring section narrow top being directed either toward or outward the planet, depending on the specially derived relationship between planetary and ring masses.
6. The solution is obtained up to second order of a small parameter but the presented method permits to obtain it up to any desired order, the first approximation being coincide with Laplace's result.
7. Kovalevskaya's manuscript of 1874 was presented by Karl Weierstrass to Goettingen University together with her two other manuscipts.

This paper was not published in time. The cause of this lies in Maxwell's theory of 1853 about dicrete rotations of particles in a ring (see above). Very possible that Maxwell's ideas were not known initially neither to Weierstrass nor Kovalevskaya at beginning of this her research and that she has found out them later, after her work finishing. Very possible is the other version: she interested mainly the potential theory solutions for fluid rotating mass and hyperelliptic fuctions conventional applications. She had found the Laplace's problem to be very attractive itself for this aim with no connection with its astronomical aspects.

1885 Eleven years later after her result of 1874, Kovalevskaya decides to publish her manuscript (see above discussion). She does it under strong and friendly insisting of her colleage Hugo Gylden (1841-1896), Director of Stockholm Royal Observatory, Fellow Editor of "Acta Mathematica", famous astronomer and mathematician. Gylden advises to publish her manuscript in german journal "Astronomische Nachrichten". He writes himself the preface to her article informing the readers that he considers the presented result to be very interesting and important for astronomy and mathematics: "Auf meine Bitte hat mir die Verfasserin erlaubt, das beifolgende Manuscript der Redaction der A.N. zu ueberreichen. Die Abhandlung ist zwar schon vor mehreren Jahren verfasst, bis jetzt jedoch nicht gedruckt worden, und so meine ich, dass die Veroeffentlichung der hochinteressanten Untersuchung den Lesern der A.N. nur angenehm sein kann. (Hugo Gylden)". In this publication of 1885, Kovalevskaya gives the short explication that she had not accomplished the series expansions up to higher orders of a small parameter because she was not sure that astronomers use Laplace's theory of Saturn rings structure till now but new theory of Maxwell: "Den Querschnitt des Ringes noch genauer zu bestimmen hat mich uebrigens, ausser der Schwierigkeit der Rechnung, der Umstand abgehalten, dass nach den Untersuchungen von Maxwell (On the Stability of the motion of Saturn's Rings, Cambridge, 1859) es zweifelhaft geworden ist, ob die Ansicht Laplace's von der Constitution der Saturnringe ueberhaupt haltbar ist" [Appendix 1, Reference 5, P.45]. Thus, her decision to not continue the expansions of series higher than up to second order concerned to her early knowledge about Maxwell's results (after this her own statement of 1885). It is not obvious whether this knowledge relates to her manuscript of 1874 too. Thus, the paper was finally edited in 1885 (see above article III in Section I and the Ref.[5] in App.1). Unfortunately, the result had no especially influence and importance for Planetary Physics because Maxwell's prediction about rings structure and nature was compatible with rings observations made in XIXth century. Nevetheless this paper turned out very soon to be some remarkable and important stage in Celestial Mechanics in framework of celestial bodies potentials theory and both elliptic and hyperelliptic functions application for the integration in stability theory of uniform fluid mass.

Kovalevskaya's paper was immediately high estimated by both Henri Poincare' and Francois Tisserand. As it was already mentioned above, the later exposed Kovalevskaya's successful results (in range of other significant and independent results of this problem) in his book [Tisserand, 1891]. Very soon her method attained the attention of specialists in hydrodynamics , especially of Horace Lamb as a useful hydrodynamical analogy [Lamb, 1932].

As is known, this problem development is connected with many others famous names of scientists. The glorious line of their names during XVII-XX centuries can be divided by three main stages of development (see below), according to Paul Appell's classification [Appell,1921,1932]

IV. Main stages of equilibrium figures theory development

From Newton to Laplace. Equilibrium figures of Solar System bodies: ellipsoids of rotation and ring-like figures [Newton, 1687 (in "Philosophiae Naturalis Principia Mathematica" considers the Earth as an uniformly rotating slightly oblate spheroid); Huygens, 1732 (in "Discours de la Cause de la Pesanteur" proves that the relative equilibrium of rotating fluid achieves when the sum of gravitation and inertia forces in any point of free surface directs normally to the surface of the isotropic body); Maupertui, 1732; Clairaut, 1737, 1743 (in "Theorie de la Figure de la Terre" considers non-isotropic bodies equilibrium); Maclaurin, 1740,1742 (generalizes Clairaut's results and proves that any oblate spheroid can be an equilibrium figure finding the relationship between angular velocity and meridional section eccentricity, Maclaurin's ellipsoids being varying from sphere to infinitely thin disc); Simpson, 1743; Laplace, 1789,1796, 1798, 1825 (proves that an uniformly rotating figure has level surfaces coinciding with surfaces of equal pressure and those of equal density); Legendre, 1789,1800,1830]

From Jacobi to Poincare'. Three-axis ellipsoids stability [Jacobi, 1834 (investigates the three-axis uniform ellipsoid); Liouville, 1834, 1843, 1846 (proves that Jacobi's ellipsoids can be equilibre only at some specified values of their angular momentum, being varying from axis-symmetrical configuration to an infinitly thin needle); Smith, 1838; Meyer, 1842; Plana, 1853; Matthiessen, 1859; Lejeune-Dirichlet, 1837,1861; Poincare', 1892,1895; Jeans, 1903,1916].

Poincare' and Lyapunov. Equilibrium stability of ellipsoidal and non-ellipsoidal bodies. Figures of Poincare' [Tompson and Tait, 1883; Liapunov (Liapunoff), 1884; Kovalevskaya, 1885 (see above); Poincare', 1885...1913 (see refs.); Schwarzschild, 1897; Hadamard, 1897; Darwin, 1902,1908,1910; Liapunov, 1903,...1925 (see refs); Steklow, 1908; Crudeli,1909,1910; Amber, 1918; Lichtenstein, 1923,1933, Nicliborc, 1929,1931,1932, Wavre, 1926, 1932].

Some of papers from this Appell's classification are mentioned in References (see below) together with some other conventional papers and books on the discussed topic. According to this formal classification, Sofia Kovalevskaya's paper of 1885 has importance in the last branch of presented Appell's three stages of theory development. The theory concernes to stability figures of equilibrium of uniform fluid structures in rotation connected mainly with results of Poincare' and Lyapunov.

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